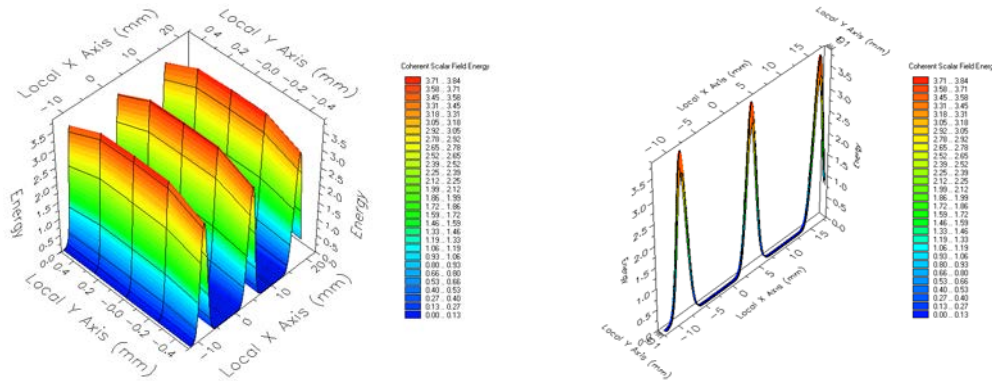


## FRED Pulse Propagation Application Note



### Introduction

Pulsed laser operation is necessary to engage nonlinear optical materials and to investigate chemistry and physics phenomena occurring in very short timeframes<sup>1</sup>. The extremely high peak power in a pulse is also important for laser ablation applications<sup>1</sup>. In this application note, the design and simulation of a desired pulse train in **FRED** is demonstrated.

### Definition of a Time-Limited Light Pulse

A pulse of finite duration can be generated by taking a summation of weighted plane waves of varying light frequencies:

$$E(t) = \frac{1}{\sqrt{2\pi}} \int E(\omega) \cdot e^{i\omega t} d\omega$$

To create a pulse with a desired duration and shape  $E(t)$ , the required spectral distribution  $E(\omega)$  can be determined with a similar relation:

$$E(\omega) = \frac{1}{\sqrt{2\pi}} \int E(t) \cdot e^{i\omega t} dt$$

The power in each spectral component (Power Spectral Density) is simply:

$$|E(\omega)|^2 = \left| \frac{1}{\sqrt{2\pi}} \int E(t) \cdot e^{i\omega t} dt \right|^2$$

The Fourier transform limit states that pulse duration and bandwidth are inversely related. Specifically,  $\Delta\omega\Delta t \geq 0.5$ . To create a very short pulse, a large range of wavelengths is required.

## Pulse Trains

A single isolated pulse requires an infinite range of wavelengths. Practical light sources have a limited bandwidth which can generate a repeating train of light pulses. By manipulating the spectral distribution of the light source, one can control the shape, width, and spacing of these pulses. To create a desired temporal spacing between pulses (Free Spectral Range  $t_{FSR}$ ), the required frequency spacing is:

$$\Delta\omega = \frac{2\pi}{t_{FSR}} = \frac{2\pi\nu}{d_{FSR}}$$

where  $\nu$ =velocity of light in the medium ( $c/n$ ) and  $d_{FSR}$  is the physical distance between pulses.

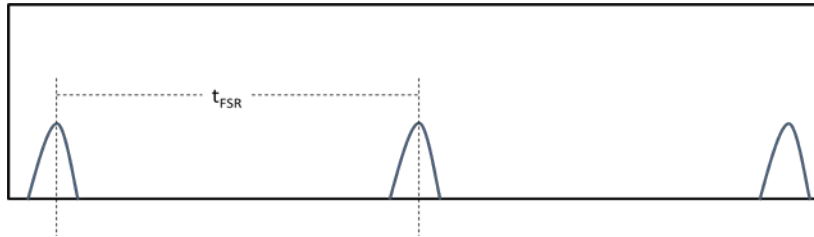


Figure 1. Pulse train with Free Spectral Range spacing.

## Example: Gaussian Pulse Train

A Gaussian pulse train can be designed and implemented in **FRED**. The steps in the design process are as follows:

1. Take the Fourier transform of the desired temporal pulse to find  $E(\omega)$ .
2. Compute the power spectral density  $|E(\omega)|^2$ .
3. Create a table of weighted light frequencies.
4. Convert frequencies into wavelengths.
5. Input into **FRED**

The desired pulse has duration (FWHM) of 10 ps and central wavelength of 1064 nm. The spacing between pulses is 12.5 mm. The pulse occurs in vacuum.

The Gaussian function has the form:

$$E(t) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}a^2t^2\right) e^{i\omega_0t}$$

where  $a$  is a constant related to the FWHM:

$$a = \frac{2\sqrt{2\ln 2}}{FWHM} = \frac{2.3548}{FWHM} = 2.3548E11 \text{ (1/s)}$$

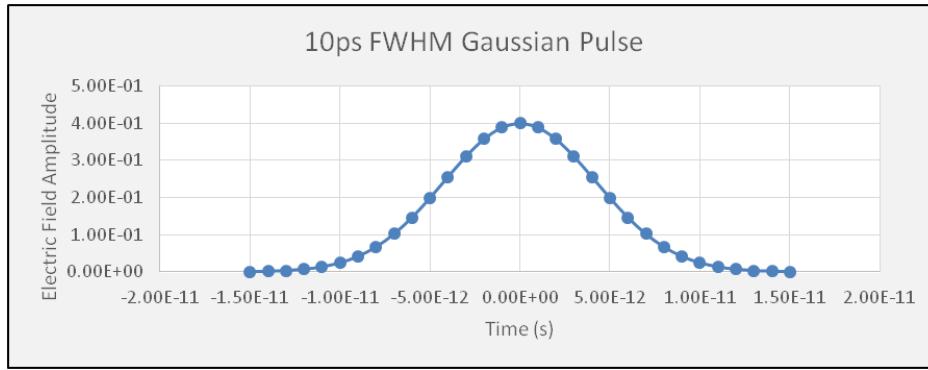


Figure 2. Temporal profile of Gaussian pulse described above.

The central frequency is computed from the central wavelength:

$$\omega_0 = \frac{2\pi v}{\lambda} = \frac{2\pi \cdot 299,792,458 \text{ m/s}}{1064E-9 \text{ m}} = 1.77035E15 \text{ Hz}$$

Frequency spacing is determined based on Free Spectral Range:

$$\Delta\omega = \frac{2\pi v}{d_{FSR}} = \frac{2\pi \cdot 299,792,458 \text{ m/s}}{12.5E-3 \text{ m}} = 1.50692E11 \text{ Hz}$$

Taking the Fourier transform of the pulse, the pulse spectrum is obtained:

$$E(\omega) = \mathcal{F}(E(t)) = \mathcal{F}\left(\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}a^2t^2\right) e^{i\omega_0 t}\right) = \frac{1}{a\sqrt{2\pi}} \exp\left(-\frac{(\omega - \omega_0)^2}{2a^2}\right)$$

The spectral power density is:

$$|E(\omega)|^2 = \left| \frac{1}{a\sqrt{2\pi}} \exp\left(-\frac{(\omega - \omega_0)^2}{2a^2}\right) \right|^2$$

The next step is to determine how many equally-spaced frequencies exist within 99% of this distribution:

$$0.01 = \left( \exp\left(-\frac{(\omega_{\max} - \omega_0)^2}{2a^2}\right) \right)^2 \quad \text{where } \omega_{\max} = \left(\frac{N-1}{2}\right)\Delta\omega + \omega_0$$

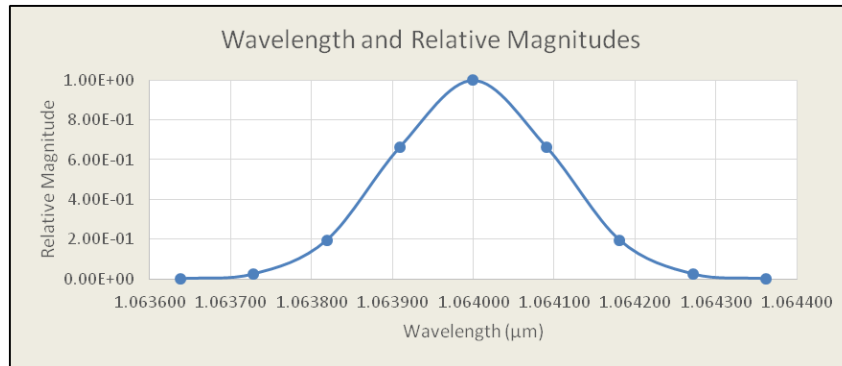
Setting the left hand equation equal to  $\omega_{\max}$ , and then setting the left hand equation equal to the right hand equation, solve for N (the number of frequencies):

$$a\sqrt{-2\ln(\sqrt{0.01})} + \omega_0 = \left(\frac{N-1}{2}\right)\Delta\omega + \omega_0$$

$$N = \frac{2a}{\Delta\omega} \sqrt{-2\ln(\sqrt{0.01})} + 1 \approx 8$$

In many cases, it is convenient to have a central wavelength, so N is increased to 9. Given the values of  $\omega_0$  and  $\Delta\omega$ , the required wavelengths and relative magnitudes are shown in the following table:

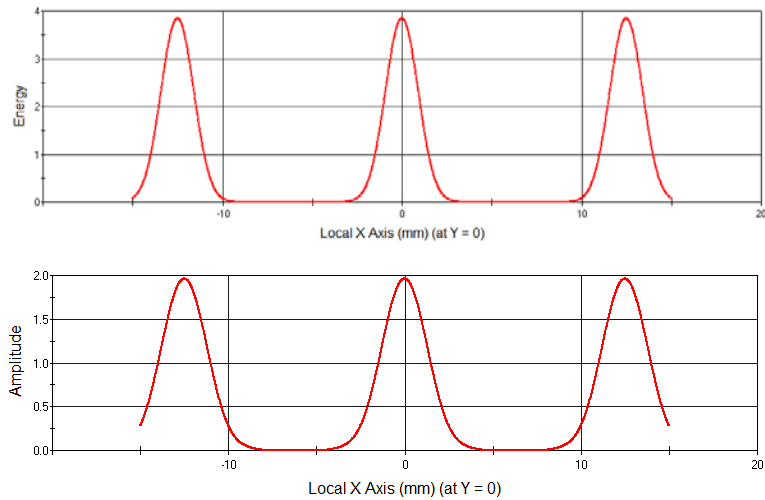
Frequency (Hz)	1.76970 E15	1.76990 E15	1.77000 E15	1.77020 E15	1.77035 E15	1.77050 E15	1.77070 E15	1.77080 E15	1.77100 E15
Wavelength (nm)	1064.362	1064.272	1064.181	1064.091	1064.000	1063.909	1063.819	1063.728	1063.638
Relative Magnitude	0.0014	0.0251	0.1944	0.6640	1.0000	0.6640	0.1944	0.0251	0.0014



**Figure 3.** Relative power spectral density of each source wavelength.

These wavelengths and amplitude values can be added to a coherent plane wave source in **FRED**. A detector with length  $>12.5$  mm oriented along the axis, allows the pulses to be plotted in space. However, interference between each wavelength component in the source cannot be modeled directly. **Different wavelength components are added incoherently by default in FRED**. To overcome this issue, an embedded script can be written to manually add the scalar field component from each wavelength across the analysis surface. The script enforces a spectral filter on the analysis surface that only allows one wavelength component to pass through at a time. Coherent addition of each wavelength's field will generate the overall pulse profile on the detector.

Figure 4 shows a cross section of the resulting pulse energy and amplitude along the optical axis. As designed, the pulse spacing is 12.5 mm. Finally, the FWHM of the pulse amplitude can be confirmed with the expected value. A pulse with temporal FWHM of 10 ps should have a spatial FWHM of  $(10E - 12s) \cdot (299,792458 m/s) = 0.003 m$ . Indeed, the FWHM of each pulse in Figure 4 is 3 mm.



**Figure 4.** Series of 3 Gaussian pulses measured along an axial detector from a coherently-added multi-spectral plane wave. A perspective view of pulse energy is shown at the beginning of the application note.

## References:

1. "Laser: Pulsed Operation." Wikipedia. Oct. 17, 2015. Accessed Nov. 3, 2015. [http://wikipedia.org/wiki/Laser#Pulsed\\_operation](http://wikipedia.org/wiki/Laser#Pulsed_operation).

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